The Solutions of three Chorographic Problems, by a Member *of the* Philoſophical Society *of* Oxford.

THE three following Problems may occur at Sea, in finding the diſtance and poſition of *Rocks, Sands,* &c. from the ſhore; or in ſurveying the Sea Coaſt; when only two objects whoſe diſtance from each other is known, can be ſeen at one ſtation: but eſpecially they may be uſeful to one, that would make a *Map* of a Countrey by a Series of Triangles deriv-ed from one or more meaſured Baſes; which is the moſt exact way of finding the bearing and diſtance of places from each other, and thence their true Lon-gitude and Latitude; and may conſequently occur to one that would in that manner meaſure a Degree on the Earth.

The Firſt Problem (Fig. 1. and 2.)

There are two objects *B* and *C*, whoſe diſtance *BC* is known, and there are two ſtations at *A* & *E*, where the objects *B*, *C* being viſible, & the ſtations one from another, the Angles *BAC*, *BAE*, *AEB*, *AEC*, are known by Obſervation, (which may be made with an ordinary Surveying *Semicircle*, or *Croſtaff*, or if the objects be beyond the view of the naked Ey, with a *Teleſcopic Quadrant*) to find the diſtances or lines *AB*, *AC*, *AE*, *EC*.

Conſtruction.

In each of the triangles *BAE*, *CAE*, two angles at *A*, *E*, being known, the third is alſo known: then take any line $\alpha\varepsilon$ at pleaſure, on which conſtitute the tri-angles $\beta\alpha\varepsilon$, $\alpha\varepsilon\gamma$ reſpectively equiangular to the trian-gles *BAE*, *AEC*; join $\beta\gamma$. Then upon *BC* conſtitnte the triangles *BCA*, *BCE* equiangular to the correſpon-

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dent triangles $\beta\gamma\alpha$, $\beta\gamma\varepsilon$, join *AE*, and the thing is manifestly done.

The Calculation.

Assuming $\alpha\varepsilon$ of any number of parts, in the triangles $\alpha\beta\varepsilon$, $\alpha\gamma\varepsilon$, the angles being given, the sides $\alpha\beta$, $\alpha\gamma$, $\varepsilon\beta$, $\varepsilon\gamma$ may be found by Trigonometry: then in the Triangle $\beta\alpha\gamma$, having the angle $\beta\alpha\gamma$, and the legs $\alpha\beta$, $\alpha\gamma$, we may find $\beta\gamma$. Then $\beta\gamma$. *BC*:: $\beta\alpha$. *BA*:: $\beta\varepsilon$. *BE*:: $\gamma\alpha$. *CA*:: $\gamma\varepsilon$. *CE*.

The second Problem (Fig. 3 & 4.)

Three objects *B*, *C*, *D*, are given, or (which is the same) the sides, and consequently angles of the triangle *BCD* are given; also there are 2 points or stations *A*, *E*, such, that at *A* may be seen the three points *B*, *C*, *E*, but not *D*; and at the station, *E*, may be seen *A*, *C*, *D*, but not *B*, that is the angles *BAC*, *BAE*, *AEC*, *AED*, (and consequently *EAC*, *AEC*,) are known by observation: to find the lines *AB*, *AC*, *AE*, *EC*, *ED*.

Construction.

Take any line $\alpha\varepsilon$ at pleasure, and at its extremitys make the angles $\varepsilon\alpha\gamma$, $\varepsilon\alpha\beta$, $\alpha\varepsilon\gamma$, $\alpha\varepsilon\delta$, equal to the correspondent observed angles *EAC*, *EAB*, *AEC*, *AED*. Produce $\beta\alpha$, $\delta\varepsilon$, til they meet in ϕ, join $\phi\gamma$; then upon *CB* describe (according to 33. 3. Eucl) a Segment of a circle, that may contain an angle $=\gamma\phi\beta$; and upon *CD* describe a Segment of a circle capable of an angle $=\gamma\phi\delta$; suppose *F* the common section of these 2 circles; join *FB*, *FC*, *FD*; then from the point *C*, draw forth the lines *CA*, *CE*, so that the angle *FCA* may be $=\phi\gamma\alpha$, and *FCE* $=\phi\gamma\varepsilon$; so *A*, *E*, the common sections of *CA*, *CE*, with *FB*, *FD*, will be the points required, from whence the rest is easily deduced.

The Calculation.

Assuming $\alpha\varepsilon$ of any number, in the triangles $\alpha\gamma\varepsilon$, $\alpha\phi\varepsilon$, all the angles being given, with the side $\alpha\varepsilon$ assum'd, ſum'd, the ſides $\alpha\gamma$, $\epsilon\gamma$, $\alpha\phi$, $\epsilon\phi$, wil be known; then in the triangle $\gamma\alpha\phi$, the angle $\gamma\alpha\phi$ with the legs $\alpha\gamma$, $\alpha\phi$ being known, the angles $\alpha\phi\gamma$, $\alpha\gamma\phi$ with the ſide $\phi\gamma$ will be known: then as for the reſt of the work in the other figure, the triangle *B C D* having all its ſides and angles known, and the angles *B FC*, *B FD*, being equal to the found $\beta\phi\gamma$, $\beta\phi\delta$; how to find *F B*, *FC*, *F D* by *Calculation* (and alſo *Protraction*) has been ſhewn by Mr *Collins* (in *Phil. Tranſact n.* 69 *p.* 2093.) as to all its caſes, which may therefore ſuperſede my ſhewing any other way.

But here it muſt be noted, that if the ſum of the obſerved angles, *B AE*, *AED*, is 180 degrees: then *AB* and *ED* cannot meet, becauſe they are parallel, and conſequently the given ſolution cannot take place; for which reaſon I here ſubjoin another.

Another Solution.

Upon *BC* (Fig. 5.) deſcribe a ſegment *B AC* of a circle, ſo that the angle of the ſegment, may be equal to the obſerved ∠ $\beta\alpha\gamma$, (which as above quoted is ſhewn 33. 3. Euclid) and upon *CD* deſcribe a ſegment *CED* of a circle, capable of an angle equal to the obſerved *C E D*; from *C* draw the diameters of theſe circles *CG*, *C H*; then upon *CG* deſcribe a ſegment of a circle *GFC*, capable of an angle equal to the obſerved ∠ *AEC*; likewiſe upon *CH* deſcribe a circles ſegment *CFH*, capable of an angle equal to the obſerved *CAE*: ſuppoſe *F* the common ſection of the two laſt circles *HFC*, *GFC*, join *FH*, cutting the circle *H E C* in *E*, join alſo *FG*, cutting the circle *G AC* in *A*: I ſay that *A*, *E*, are the points required.

Dem :

For the ∠ *BAC* is = $\beta\alpha\gamma$ by conſtruction of the ſegment, alſo the angles *CEH*, *CAG*, are right, becauſe each exiſts in a ſemicircle: therefore a circle being deſcribed upon *CF* as a diameter, wil paſs thro *E*, *A*; Therefore

fore the angle $CAE = \angle CFE = CFH =$ (by construction) to the observed angle $\gamma\alpha\varepsilon$. In like manner the $\angle CEA = CFA = CFG =$ observ'd angle $\gamma\varepsilon\alpha$.

If the stations A, E fall in a right line with the point C; the lines GA, HE being parallel, cannot meet: but in this case the problem is indeterminate and capable of infinite Solutions. For as before upon CG describe a Segment of a circle capable of the observed $\angle\gamma\varepsilon\alpha$, and upon CH, describe a Segment capable of the observed $\gamma\alpha\varepsilon$: then thro C, draw a line any way cutting the circles in A, E, these points wil answer the question.

The Third Problem.

Four points B, C, D, F, (Fig. 6.) or the 4 sides of a quadrilateral, with the angles comprehended are given; also there are 2 stations A and E such, that at A, only B, C, E are visible, and at E only A, D, F, that is, the angles BAC, BAE, AED, DEF are given: to find the places of the two points A, E; and consequently, the lengths of the lines AB, AC, AE, ED, EF.

Construction.

Upon BC (by 33. 3.Eucl.) describe a segment of a circle, that may contain an angle equal to the observed angle BAC, then from C draw the chord CM, or a line cutting the circle in M, so that the angle BCM may be equal to the supplement of the observed angle BAE, i. e. its residue to 180 degrees. In like manner on DF describe a segment of a circle, capable of an angle equal to the observ'd DEF, and from D draw the chord DN, so that the angle FDN may be equal to the supplement of the observ'd angle AEF, join MN, cutting the 2 circles in A, E: I say A, E, are the two points requir'd.

Dem:

Join AB, AC, ED, EF, then is the $\angle MAB = \angle BCM$ (by 21. 3. Eucl:) = supplement of the observ'd $\angle BAE$ by construction, therefore the constructed

$\angle BAE$

∠ BAE is equal to that which was obſerved. Alſo the ∠ BAC of the ſegment, is by conſtruction of the Segment, equal to the obſerv'd ∠ BAC. In like manner the conſtructed angles AEF, and DEF, are equal to the correſpondent obſerved angles AEF, DEF, therefore A, E are the points required.

The Calculation.

In the Triangle BCM, the ∠ BCM (= ſupplement of BAE) and ∠ BMC (= BAC) are given, with the ſide BC; thence MC may be found; in like manner DN in the △ DNF may be found. But the ∠ MCD (= BCD -- BCM) is known with its legs MC, CD, therefore its baſe MD, and ∠ MDC may be known. Therefore the ∠ MDN (= CDF -- CDM -- FDN) is known, with its legs MD, DN; thence MN with the angles DMN, DNM, will be known. Then the ∠ CMA (= ∠ DMC + DMN) is known, with the ∠ MAC (= MAB + BAC) and MC before found; therefore MA and AC wil be known. In like manner in the triangle EDN. the angles E, N, with the ſide DN being known, the ſides EN, ED will be known; therefore AE (= MN -- MA -- EN) is known. Alſo in the triangle ABC, the ∠ A with its ſides BC, CA being known, the ſide AB will be known, with the ∠ BCA; ſo in the triangle EFD, the ∠ E with the ſides, ED, DF being known, EF wil be found, with the ∠ EDF. Laſtly in the triangle ACD, the ∠ ACD (= BCD -- BCA) with its legs AC, CD being known, the ſide AD wil be known, and in like manner EC in the triangle EDC.

Note that, in this problem, as alſo in the firſt and ſecond, if the two ſtations fall in a right line with either of the given objects: the *locus* of A, or E, being a circle, the particular point of A, or E, cannot be determined from the things given.

As to the other caſes of this third problem, wherein A, and E, may ſhift places, i. e. only D, F, E may be viſible at *A*, and only *A, B, C* at *E*; or wherein *B, D, E*, may be viſible at *A*, and only *C, F, A*, at *E*; or wherein *A* may be of one ſide of the quadrilateral, and *E* on the other; or one of the ſtations within the quadrilateral, and the other without it: I ſhall for brevitys ſake omit the figures, and diverſity of the Signs + and -- in the calculation, and preſume that the Surveyour wil eaſily direct himſelf in thoſe caſes, by what has been ſaid.

The Solution of this third problem is general, and ſerves alſo for both the precedent. For ſuppoſe *C, D* the ſame point in the laſt figure, and it gives the ſolution of the ſecond problem: but if *B, C*, be ſuppos'd the ſame points with *D, F*, by proceeding as in the laſt, you may directly ſolve the firſt problem.

A Letter from William Molyneux *Eſq; to one of the* Secretarys *of the* R. S. *concerning the Circulation of the blood as ſeen, by the help of a Microſcope, in the* Lacerta Aquatica.

Dublin Octob. 27. 1685.

Sir,

OUR Society lately received tranſcripts of two of D^r^ *Gardens* Letters, the firſt dated form *Aberdeen* July 17. 1685. to D^r^ *Middleton*; the other Sept. 4. 1685. to D^r^ *Plot*. To both theſe Letters I have ſomething to ſay.

In the firſt he gives an Account of the Viſible Circulation of the blood in the *Water-Newt* or *Lacerta Aquatica*; truely I am heartily glad, that this Learned and Ingenious D^r^ has hit upon this Experiment; tis now above two years and an half, ſince I firſt Diſcovered this ſurpriſing

Fig. 1.

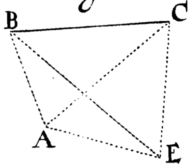


Fig. 2.

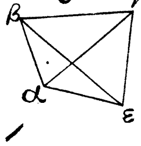


Fig. 3.

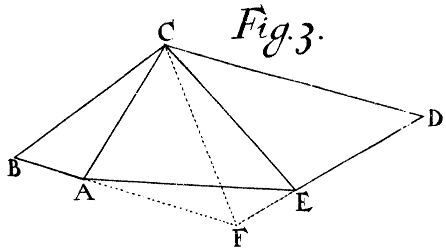


Fig. 4.

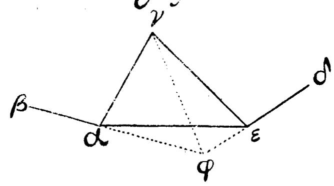


Fig. 5.

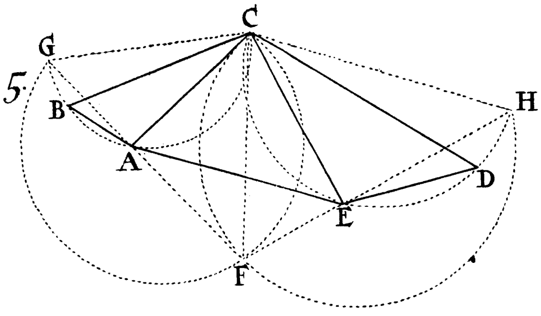


Fig. 6.

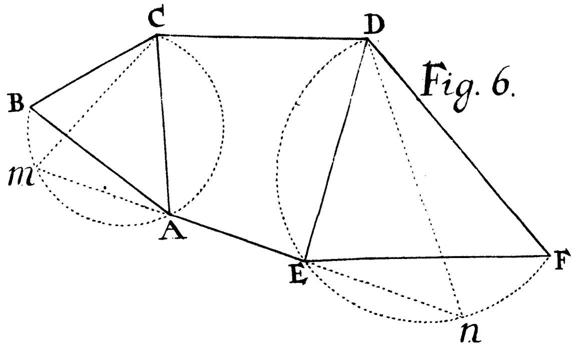
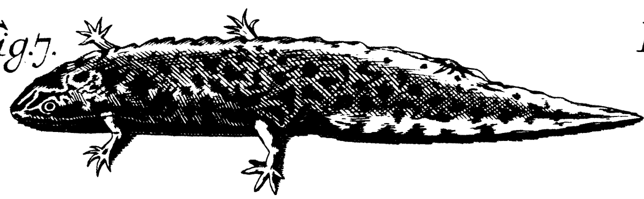


Fig. 7.



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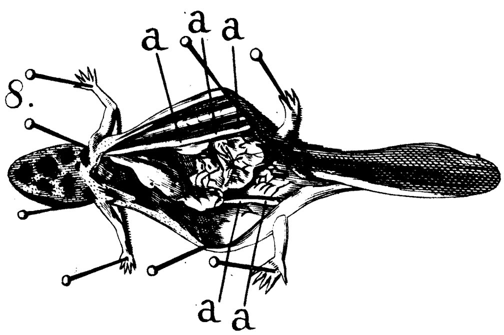

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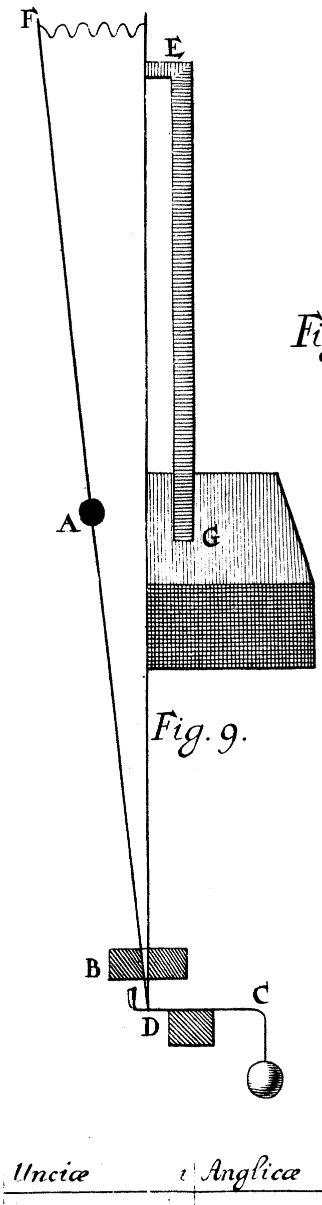

Fig. 10.

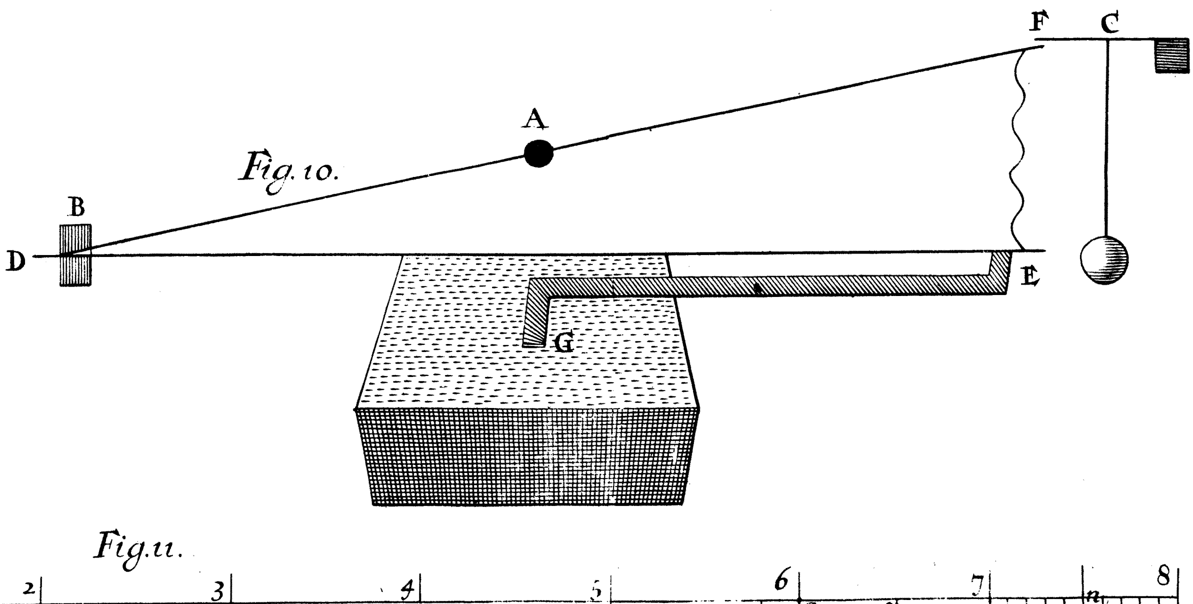

Fig. 11.